Example 3

At What Inclination Angle do Candles Burn the Fastest?

Word Count: 1654

Table of Content

Example 3 (continued)

Statement of the Task

In this mathematical investigation, the main aim is to explore and develop the different mathematical concepts involved in the burning of candles. There are many such concepts, for instance concerning the temperature or the weight of candles. More specifically I chose to investigate at which various angles candles burned the fastest. To do so I tested three identical candles at four different angles. With this data I was able to draw conclusions concerning the angle at which a candle burns the fastest and the slowest. Furthermore, this investigation involves several mathematical aspects especially geometrically. All of these will be examined later in the project. Finally, since all the candles are of the same length, I chose to explore at which stage of the consumption the candle burns the fastest. With that data, it was possible to make up a cumulative consumption curve for each candle. Results will be analyzed and explained using graphs, tables and illustrations. The outcome should be a strong mathematical understanding on why a candle burns fastest at a certain angles and why the shape of its flame is also important.

Data Collection

Before the investigation, some information concerning the candles needed to be recorded. For instance the weight of the candle is needed to be able to calculate how much of the candle was consumed. Also I recorded the length of the candles in order to be able to formulate a cumulative consumption curve. Also each candle was of a diameter of 1cm, thus the volume of the candle (if we consider each candle as a perfect cylinder) is $\prod 0.5^2 \times 21.2$, which is equal to 16.7cm^3. Later this will enable use to calculate the total volume of wax consumed in each candle.

Candle	Original Weight (g)			
	90°	45°	0°	315°
1	60.78	60.56	60.87	60.65
2	60.34	60.85	60.87	60.43
3	59.87	61.01	60.54	59.96

Candle	Original Length (cm)			
	90°	45°	0°	315°
1	21.2	21.2	21.2	21.2

Example 3 (continued)

2	21.2	21.2	21.2	21.2
3	21.2	21.2	21.2	21.2

The first investigation consisted in burning three identical candles standing at a 90° angle. Then we repeated the processes at 45°, 0° and 315°. The weight and length of the candles were recorded after 1 hours of burning. The results showed this:

Candle	Weight (g) after 1 hour burning			
	90°	45°	0°	315°
1	50.67	43.86	24.09	59.42
2	50.87	44.65	24.78	60.21
3	49.56	44.51	24.54	59.02

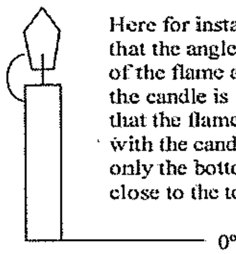
Candle	Length (cm) after 1 hour burning			
	90°	45°	0°	315°
1	13.8	9.2	4.5	21.1
2	13.7	9.3	4.4	21.1
3	13.8	9.3	4.2	21.1

In order to formulate a cumulative consumption curve I marked each candle every centimeter over a length of eight cm. The time of consumption of the candle was recorded every centimeter. After the candles were burned I could figure out what length of the candle was consumed and the volume of wax consumed. The results showed up as follows:

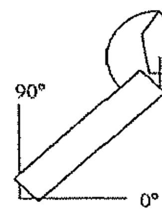
Angle	90°			45°			0°			315°		
	Candle											
Time to consume: (in min)	1	2	3	1	2	3	1	2	3	1	2	3
1 cm	8:01	8:03	7:52	5:21	5:31	5:23	3:32	3:37	3:31	-	-	-
2 cm	15:45	15:50	15:23	10:45	10:54	10:43	7:09	7:14	7:05	-	-	-
3 cm	23:34	23:43	22:55	16:03	16:09	16:04	10:34	10:39	10:32	-	-	-
4 cm	31:00	31:13	32:42	21:25	21:28	21:28	14:06	14:12	14:01	-	-	-
5 cm	39:23	39:45	39:02	26:40	26:42	26:41	17:39	17:45	17:31	-	-	-
6 cm	47:34	47:54	47:12	31:58	32:04	31:56	21:07	21:14	21:00	-	-	-
7 cm	55:29	55:38	55:21	37:15	37:23	37:15	24:43	24:53	24:40	-	-	-
8 cm	-	-	-	42:37	42:46	42:35	28:17	28:22	28:31	-	-	-

One last geometrical aspect that we need to observe is the behavior of the flame and the candle when the angle of inclination is changed.

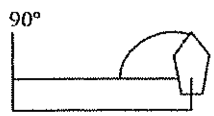
Example 3 (continued)



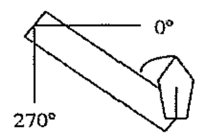
Here for instance we can see that the angle between the top of the flame and the bottom of the candle is 180°. This meaning that the flame is in straight line with the candle. As a result, only the bottom of the flame is close to the top of the candle.



When the inclination of the candle is 45°, the flame does not change position. As a result the angle between the top of the flame and the bottom of the candle decreases to 135°. The result is that more of the candle is comes closer to the flame.



When the candle is at a 0° angle, the angle is reduced even further. It now becomes a 90° angle. Most of the top of the candle is now in contact with the flame.



Finally, if the candle is inclined at a 315° angle, the angle becomes a 45° angle, and the whole flame is in contact with the candle.

This geometrical behavior will be important in the analysis of the consumption rate since it plays an important role in the speeding up of the burning process.

Analysis of Results

Using all the data collected throughout the different investigations I was able to make several observations and calculate changes that occurred in volume, size and mass. Furthermore I was able to graph some of the data into charts. The main conclusions that I was able to draw were concerning the different burning rates at different angles, the changes in volume and mass, as well as the cumulative behavior of the burning rates. These results will help me conclude at which angle a candle burns the fastest.

Changes in Weight

The results of the different tests made at different angles showed interesting features concerning the speed of consumption. The results clearly show that candles that are

Example 3 (continued)

inclined at an angle between 0°-90° or 90°-180° burn faster than candles that a straight at an angle of 90°. The reason why this conclusion can be made is because the changes in weight between the candles, which clearly show that candles that a slanting are lighter than candles that are standing, after 1 hour of burning.
Here are some calculations that prove this:

At 90°, the average weight of the candles is:
(60.78 + 60.34 + 59.87) / 3 = 60.3g
After they have burned for 1 hour their average weight is:
(50.67 + 50.87 + 49.56) / 3 = 50.4g
So the average change in weight is:
60.3 - 50.4 = 9.9g
So at a 90° angle, an average of 9.9g of candle was consumed in 1 hour.
This means 16.4% of the original candle was consumed. This is found as follows:
9.9 / 60.3 x 100 = 16.4

At 45°, the average weight of the candles is:
(60.56 + 60.85 + 61.01) / 3 = 60.8g
After they have burned for 1 hour their average weight is:
(43.86 + 44.65 + 44.51) / 3 = 44.3g
So the average change in weight is:
60.8 - 44.3 = 16.5g
So at a 90° angle, an average of 16.5g of candle was consumed in 1 hour.
This means 27.1% of the original candle was consumed. This is found as follows:
16.5 / 60.8 x 100 = 27.1

At 0°, the average weight of the candles is:
(60.87 + 60.87 + 60.54) / 3 = 60.8g
After they have burned for 1 hour their average weight is:
(24.09 + 24.78 + 24.54) / 3 = 24.4g
So the average change in weight is:
60.8 - 24.4 = 36.4g
So at a 0° angle, an average of 36.4g of candle was consumed in 1 hour.
This means 59.9% of the original candle was consumed. This is found as follows:

Example 3 (continued)

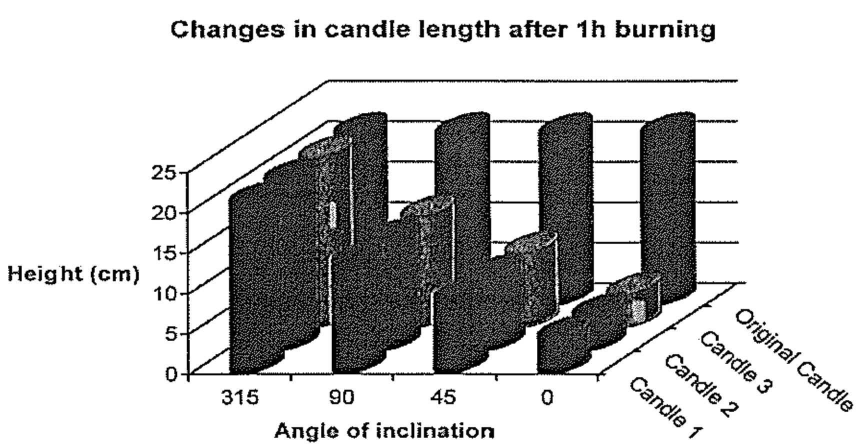
36.4 / 60.8 x 100 = 59.9

At 315°, the average weight of the candles is:
(60.65 + 60.43 + 59.96) / 3 = 60.3g
After they have burned for 1 hour their average weight is:
(59.42 + 60.21 + 59.02) / 3 = 59.6g
So the average change in weight is:
60.3 – 59.6 = 0.7g
So at a 315° angle, an average of 0.7g of candle was consumed in 1 hour.
This means 1.2% of the original candle was consumed. This is found as follows:
0.7 / 60.3 x 100 = 1.16

The results prove that more wax is consumed when the candles are slanted, except for the candle at a 315° angle, which did not work because the hot melting wax turned off the candle at such an angle.

Changes in Length

Another way of investigating at which angle a candle consumes the fastest is by using the changes in candle length at different inclination angles. Here is a graph showing these changes:



Example 3 (continued)

As is shown in this chart, the candles burned with an inclination angle of 45° or 0° burned the fastest and as a result these candles were much shorter than the original. As a matter of fact, at 90° only 35% of the original length was consumed, at 45°, 56.6% were burned, and at 0°, 79.2% were consumed.

Changes in Volume

If the candles are viewed as perfect cylinders, then the volume of wax consumed in each investigation can be calculated. These changes in volume also support the thesis that candles burn fastest when they are slanted.
For instance here are some calculations that show these changes:

Original volume of all candles is $16.7cm^3$.
At 90° inclination angle, after 1 hour of burning the volume of wax left is:
$\prod 0.5^2 \times 13.8 = 10.8cm^3$
This means that 5.9 cm^3 of wax were consumed in one hour.

At 45° inclination angle, after 1 hour of burning the volume of wax left is:
$\prod 0.5^2 \times 9.3 = 7.3cm^3$
This means that 9.4 cm^3 of wax were consumed in one hour.

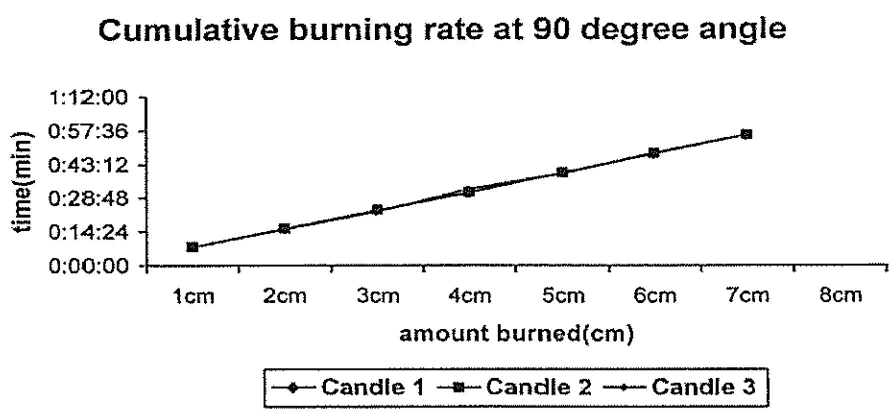
At 0° inclination angle, after 1 hour of burning the volume of wax left is:
$\prod 0.5^2 \times 4.4 = 3.45cm^3$
This means that $13.25cm^3$ of wax were consumed in one hour.

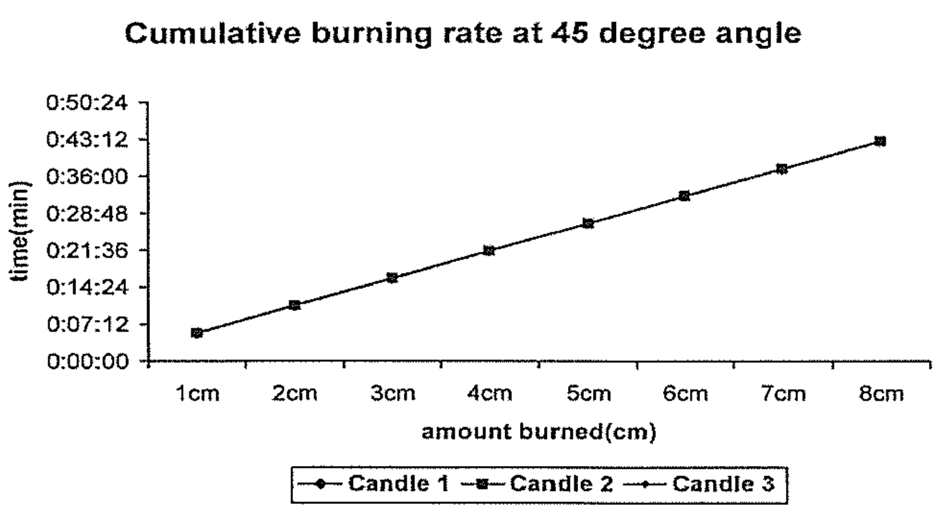
Here again the results support the idea that a candle that is slanted consumes more wax in the same amount of time as a candle standing straight.

Example 3 (continued)

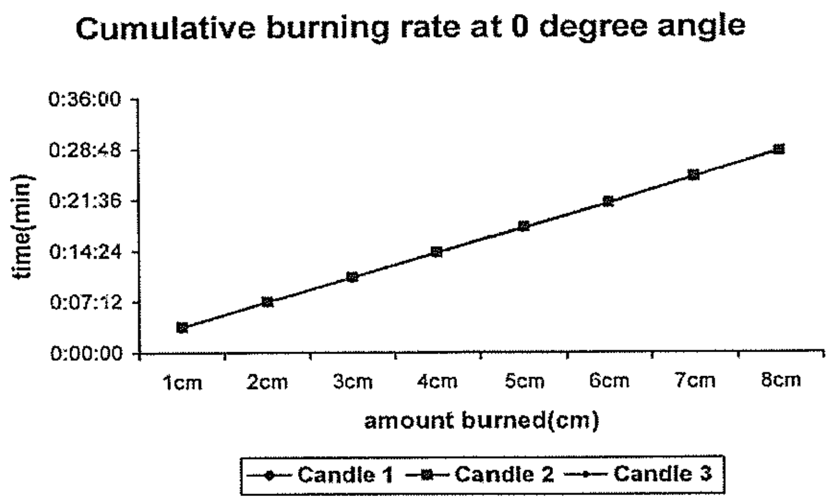
Cumulative Consumption

The point of this investigation was to figure out which phase during the hour of burning is the fastest. Also with this data, the speed of the consumption can be calculated. First of all, here are the cumulative curves of the different candles:





Example 3 (continued)



These graphs show that the consumption rate of wax is constant at all angles. Changes in angles do not create an increase or a decrease in the rate of consumption.

Conclusion

By using different mathematical approaches, I was able to conclude upon many aspects of the investigation. Using differences in weight, height and volume as well as differences in the cumulative burning rates, I was able to show that at an angle of inclination below or beyond 90°, the burning rate speeds up. However this investigation also proved that passed a certain angle the melting wax turns of the candle, and the candle will not burn.

Even though the investigation contains many possible errors, the results are generally so constant that one can say they are more or less reliable. Possible errors could have been made when measuring time, or lengths as well as weights. These could have possibly resulted in slightly different results but they would not have affected the possible conclusions made from this investigation.

Example 3

At what inclination angle do candles burn the fastest?

Assessment

Criterion	A	B	C	D	E	F	G	Total
Achievement level awarded	2	3	2	2	1	2	1	13
Maximum possible achievement level	2	3	5	3	2	3	2	20

Moderator's comments

Criterion A: Introduction

The project has a title, a clear statement of the task and a description of the plan.

Criterion B: Information/measurement

The data generated is relevant and organized ready for use and is just sufficient in quantity.

Criterion C: Mathematical processes

The mathematical processes used are simple and accurate. The vertical scales on the "cumulative burning rate" graphs are different and this may pose problems for the analysis.

Criterion D: Interpretation of results

The student has provided at least one interpretation that is consistent with the processes used.

Criterion E: Validity

The student has made an attempt to comment on the conclusions made but not on the processes used.

Criterion F: Structure and communication

The communication is not always clear but an attempt has been made to structure the project. The notation and terminology are consistent throughout.

Criterion G: Commitment

Satisfactory commitment is evident.